$$1-\frac{3}{2}\,\frac{B}{D_\infty}\,\frac{\sin\varphi_0}{\sin\varphi_\infty}+\frac{B^3}{2D_\infty^3}\left(\frac{\sin\varphi_0}{\sin\varphi_\infty}\right)^3=0$$

in Eq. (7). We then obtain

$$\sin\varphi_\infty=\frac{B\mathrm{Ca}}{Q}\sin\varphi_0. \tag{10}$$

Consequently, at angles $\sin\varphi < \sin\varphi_\infty$ the film thickness remains uniform in every case. We note that when $\varphi_0 \geqslant 90°$ and $B\sin\varphi_0 \leqslant D_0 \leqslant B$, the rate of fluid flow Q is determined from the relation (5), and expression (10) becomes

$$\sin\varphi_\infty=\frac{B}{D_0}\sin\varphi_0.$$

For determining the film thickness D around the circumference of a rotating drum, therefore, one must use expressions (1)-(4), calculate the thickness D_0 of the film entrained by a flat plate pulling out at the angle φ_0, then from either relation (5) or relation (6) find the rate of fluid flow Q, and finally from expressions (7)-(10) determine the film thickness as a function of the angle φ.

NOTATION

x, y, Cartesian coordinates; τ_0, yield point; μ_p, plastic viscosity; ρ, density; g, acceleration due to gravity; p, pressure; σ, surface tension; and U, linear velocity of the drum surface.

LITERATURE CITED

1. S. Tharmalingam and W. L. Wilkinson, "The coating of Newtonian liquids onto a rotating roll," Chem. Eng. Sci., 33, 1481-1487 (1978).
2. Z. P. Shul'man and V. I. Baikov, Rheodynamics of and Heat and Mass Transfer during Film Flow [in Russian], Nauka i Tekhnika, Minsk (1979).

COALESCENCE OF CONCENTRATED FINE-DISPERSE EMULSIONS DURING TURBULENT STIRRING

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The coalescence of droplets due to an average gradient of velocity fluctuations is considered and its dependence on the concentration of the dispersed phase is analyzed within the framework of the theory of locally isotropic turbulence.

The rate of separation of unstable emulsions which have formed during concurrent motion and mixing of mutually insoluble fluids determines the effectiveness of a great many technological processes in the chemical, petroleum, food, and various other industries. The enlargement of droplets of the dispersed phase during turbulent flow of unstable emulsions through pipelines makes it possible to increase the productivity of sedimentation and extraction equipment [1].

An analysis of the interaction between fine-disperse droplets during the flow of emulsions also facilitates the solution of problems pertaining to two-phase flow through pipelines. The true contents of each phase, the limits of existence of various stream structures, and transitions from one structure to another depend largely on the coalescence of droplets, and this dependence affects the hydrodynamics of a two-phase stream as well as the heat and mass transfer in it.

The object of this study will be to determine how the coalescence of droplets during turbulent flow of an emulsion through a pipeline depends on the concentration of the dis-

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persed phase. The average dimension of droplets will be assumed not to exceed the microscale of turbulent fluctuations. This will, first of all, make it permissible to consider only the gradiental (viscous) mechanism of coalescence. Secondly, according to available data [2], such emulsions behave like one-phase Newtonian fluids over a rather wide range of conditions.

On the basis of the last assumption, it is possible to evaluate, within the framework of the theory of locally isotropic turbulence [3], the reverse effect of the concentration of dispersed droplets on the level of their pulsation velocities. A turbulent stream can, moreover, be characterized by the amount of energy dissipation, which is ultimately due to the viscosity of the two-phase system. This then determines in what way the rheological characteristics of an emulsion reflect that the coalescence of droplets in a turbulent stream depends on their concentration.

1. Coalescence Frequency of Fine-Disperse Droplets. The transport of droplets colliding due to turbulent stirring is effected through velocity fluctuations. It is well known that mutual stirring of fluids in volumes which are large relative to the turbulence microscale cannot result in collisions between particles which are small relative to this scale [4]. Consequently, the mechanism of interaction between fine-disperse droplets is determined principally by the average gradient of fluctuation velocities.

Under these conditions, according to another study [5], additional introduction of a collision effectiveness index which shows what fraction of all possible collisions results in coalescence will lead to the expression

$$\theta = \frac{4}{3} K_g n \delta^3 G. \tag{1}$$

Here the number of droplets in a unit volume of emulsion can be expressed through the relative volume content $W = \pi n \delta^3/6$ of the dispersed phase, assuming a monodispersity of droplets. The average gradient of fluctuation velocities in a locally isotropic turbulent stream is [6]

$$G = \sqrt{2\varepsilon/15\nu_e}. \tag{2}$$

The frequency of coalescence of droplets colliding due to stirring of an emulsion with a paddle wheel designed so as to satisfy the assumption of local isotropy in a turbulent stream was studied experimentally [7]. Visual observations in this study established that, under the conditions of the experiment, the scale of turbulent vortices was much larger than the size of droplets. Despite this evidence, the authors unjustifiably matched their data against various expressions for the frequency of collisions due to velocity fluctuations between large droplets with dimensions exceeding λ_0 and incorrectly concluded that these data were consistent. Calculation on the basis of relation (1) yielded a collision frequency of 1.9 $\sec^{-1}$, taking into account the average energy dissipation under the given conditions [7], while the corresponding real collision frequencies were 1.3 and 3.1 $\sec^{-1}$. This confirms the validity of relation (1) for calculating the coalescence frequency when $K_g = 0.02$ [7].

A turbulent stream of a fine-disperse emulsion in a pipeline is characterized, on the basis of available data [2], by energy dissipation in accordance with the energy balance which, for an element of the fluid, can be expressed as $\varepsilon = \lambda_e U_0^3/2D$. The final expression for the coalescence frequency of droplets is

$$\theta = 0.01315 W \sqrt{\lambda_e U_0^3/D\nu_e}, \tag{3}$$

its distinctive feature being a nonlinear relation between this frequency and the concentration of the dispersed phase through λ_e and ν_e.

2. Viscosity of Fine-Disperse Emulsions. The effective viscosity of emulsion systems does not depend on the concentration of the disperse phase uniquely. This is reflected in the fact that in emulsions of components with different viscosities and different amounts of surfactants or contaminants there form different interphase adsorption layers, which results in different effective viscosities at a constant ratio of continuous-phase and dispersed-phase volumes [8]. Therefore, individual differences in rheological characteristics of emulsions, in this case evaluated in terms of the effective viscosity, are a major factor complicating a study of coalescence which takes into account the volume ratio of phases.

For a quantitative evaluation of the dependence of the coalescence rate of droplets in concentrated emulsions on the effective viscosity, we will use the experimentally established relation [8]

$$\mu_e = \mu_c \exp\left(\frac{K_R W}{1 - W}\right), \qquad (4)$$

where the constant K_R is defined by the expression $2.5(0.4 + \mu_d^*/\mu_c)/(1.0 + \mu_d^*/\mu_c)$, and the effective viscosity of the dispersed phase is $\mu_d^* = \mu_d + \mu_s$. The second term represents the contribution of surface active additives and interphase adsorption layers to the viscosity characteristics of emulsions.

A linear dependence of ln (μ_e/μ_c) on the ratio $W/(1 - W)$ was established experimentally through testing of twelve different unstable emulsions of organic fluids with the volume fraction of the dispersed phase not exceeding 0.33 [8]. These emulsions were, moreover, characterized by a Newtonian behavior. The viscosity of their continuous phase was varied from 21.7 to 402 cP, the viscosity of their dispersed phase was varied from 0.63 to 31 cP. The variation of K_R was confined within the interval from 1 to 2.5, closely corresponding to theoretical concepts and experimental data [9].

For some emulsions such as those formed by mixtures of light hydrocarbons in water, however, this relation follows the above trend as the concentration of the dispersed phase continues to be increased up to 0.47 [2]. Experimental data on the effective viscosity of water—naphtha emulsions [10] are presented in Fig. 1, these data suggesting that for model calculations the relation (4) remains valid up to W = 0.5. We will subsequently assume, therefore, that the concentration dependence of the effective viscosity of fine-disperse emulsions with $0 \leqslant W \leqslant 0.5$ indicates the presence of surface active additives according to relation (4) in real systems where $1 \leqslant K_R \leqslant 2.5$. The subsequent analysis will be confined to systems which do not exhibit anomalous non-Newtonian characteristics.

3. Calculation of the Coalescence. On the basis of the volume balance in the dispersed phase and considering the change in the number of droplets upon their coalescence due to turbulent stirring during motion through a pipeline, there has been obtained the system of differential equations [11]

$$\frac{d\delta}{dl} = -\frac{\delta}{3n}\frac{dn}{dl}; \quad \frac{dn}{dl} = -\frac{1}{2}\frac{\theta n}{U_0}. \qquad (5)$$

The relation between the length of the pipeline and the attained degree of enlargement δ/d_0 is obtained by integrating this system of equations with respect to the droplet diameter, with the boundary condition $\delta|_{l=0} = d_0$, viz.

$$l = \frac{6U_0}{\theta}\ln\left(\frac{\delta}{d_0}\right). \qquad (6)$$

In order to establish the level of energy dissipation, we express U_0 through the pressure drop according to the Darcy—Weissbach law and with the emulsion regarded as a homogeneous fluid characterized by effective values of parameters [2] which take into account the concentration of the dispersed phase:

$$\frac{\Delta p}{\Delta l} = \lambda_e \frac{\rho_e U_0^2}{2D}. \qquad (7)$$

The effective hydraulic drag coefficient is defined according to the Blasius law with $Re_e = U_0 D \rho_e/\mu_e$, where μ_e is defined by relation (4) and $\rho_e = \rho_d W + \rho_c(1 - W)$.

In this way, the length of a pipeline necessary for enlarging the mean dimension of monodisperse droplets in concentrated emulsions from d_0 to $d_L < \lambda_0$ can be calculated with the aid of expressions (6), (7), and (3).

Typical results of calculations for the coalescence frequency of droplets and the length of a pipeline necessary for enlarging the droplets of the dispersed phase from 10 to 100 μm are shown in Figs. 2 and 3, with the parameters D = 0.2 m, μ_c = 0.02 Pa·sec, ρ_c = 866 kg/m³, ρ_d = 1163 kg/m³, and $\Delta p/\Delta l$ = 900 Pa/m. The concentration of the dispersed phase has been taken into account according to the exponential relation (4) and the expression for the energy dissipation in a locally isotropic turbulent stream, both determining the intensity of interaction between droplets smaller than the microscale of velocity fluctuations.

It has been found that in the range of concentrations up to 10% the coalescence frequency of droplets increases linearly and the average velocity gradient in a turbulent

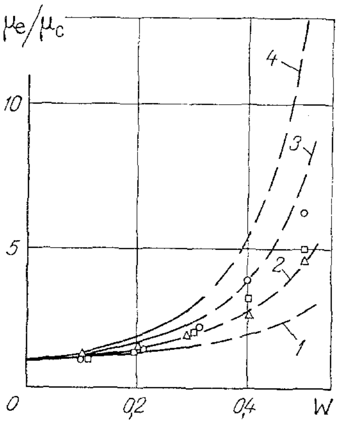

Fig. 1

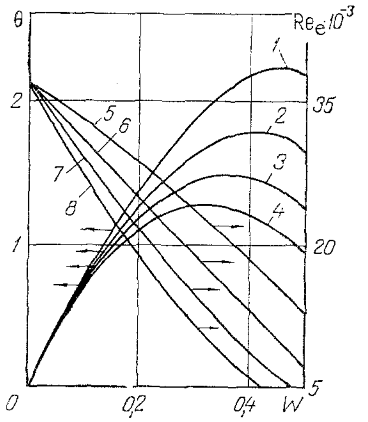

Fig. 2

Fig. 1. Dependence of the dimensionless effective viscosity μ_e/μ_c on the volume fraction of the dispersed phase W: 1) $K_R = 1.0$; 2) $K_R = 1.5$; 3) $K_R = 2.0$; 4) $K_R = 2.5$, according to the data in study [8]; dots represent experimental values [10].

Fig. 2. Dependence of the coalescence frequency θ ($\sec^{-1}$) of droplets (curves 1-4) and of the Reynolds number Re_e of the emulsion (curves 5-8) on the volume fraction W of the dispersed phase: 1, 5) $K_R = 1.0$; 2, 6) $K_R = 1.5$; 3, 7) $K_R = 2.0$; 4, 8) $K_R = 2.5$.

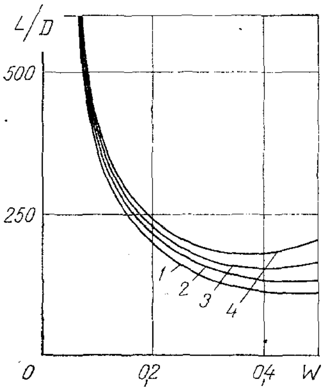

Fig. 3. Dependence of the dimensionless pipeline length L/D on the volume fraction W of the dispersed phase, for a constant degree of enlargement $d_L/d_0 = 10$ ($d_0 = 10$ μm). 1) $K_R = 1.0$; 2) $K_R = 1.5$; 3) $K_R = 2.0$; 4) $K_R = 2.5$.

stream hardly depends on the concentration at all. At volume fractions $W > 0.15$, however, the dependence of the coalescence frequency on it departs from a linear one. As the volume fraction W increases further, the relative change of θ becomes comparable with that due to the reverse effect of the concentration, according to which it decreases because of a decreasing average velocity gradient. This corresponds to peak on the curves in Fig. 2, beyond which the coalescence frequency decreases despite the continuing increase of W. This peak is in many ways determined by the individual physicochemical properties of emulsions, and in our case, with K_R changing over the range from 1 to 2.5, it corresponds to a 0.3-0.47 volume fraction of the dispersed phase.

Analogous features characterize the dependence of the pipeline length necessary for attaining a given enlargement of droplets on the concentration of the dispersed phase within a selected range. Calculations were made based on the semi-phenomenological model of coalescence in a turbulent stream. We will note, additionally, that increasing W to 0.3 accelerates the coalescence of droplets appreciably, whereupon the coalescence rate stabilizes, and with W exceeding 0.3-0.4 it begins to decrease.

On the whole, the effective viscosity with the constant K_R is the main factor determining the trend of the concentration dependence as shown in Figs. 2 and 3 within the framework of the theory of locally isotropic turbulence.

NOTATION

λ_0, turbulence microscale; K_g, collision effectiveness index; K_R, rheological constant; θ, coalescence frequency of droplets; n_0 and n, respectively, initial and the current number of droplets in a unit volume; G, average gradient of fluctuation velocities; W, volume fraction of the dispersed phase; ε, mean energy dissipation per unit mass of a turbulent stream per unit time; ν_e, effective kinematic viscosity of the emulsion; λ_e, effective hydraulic drag coefficient; U_0, mean-rate stream velocity; D, pipeline diameter; μ_c, μ_d, μ_e, μ_s, dynamic viscosity of the continuous phase, the dispersed phase, the emulsion, and the interphase boundary layer; $\Delta p/\Delta l$, pressure drop along the pipeline per unit length; d_0, δ, d_L, initial, the current, and the maximum diameter of droplets; Re_e, Reynolds number referred to the effective values of emulsion parameters; ρ_c, ρ_d, ρ_e, density of the continuous phase, the dispersed phase, and the emulsion; l and L, respectively, current and the necessary pipeline length for enlargement of droplets to a given degree.

LITERATURE CITED

1. K. Hanson (ed.), Latest Achievements in Fluid Extraction [Russian translation], Khimiya, Moscow (1974).
2. J. P. Ward and J. C. Knudsen, "Turbulent flow of unstable liquid–liquid dispersion: drop sizes and velocity distribution," J. AIChE, 13, No. 2, 356-365 (1967).
3. L. D. Landau and E. M. Lifshits, Mechanics of Continuous Media [in Russian], GITTL, Moscow (1953).
4. V. G. Levich, "Theory of colloid coagulation in a turbulent stream of a fluid," Dokl. Akad. Nauk SSSR, 99, No. 5, 809-812 (1954).
5. Y. Argaman and W. J. Kaufman, "Turbulence and flocculation," Proc. ASCE, 96, SA 2, 223-241 (1970).
6. G. Conte-Bello, Turbulent Flow through a Channel with Parallel Walls [Russian translation], Mir, Moscow (1968).
7. J. Y. Park and L. M. Blair, "The effect of coalescence on the drop size distribution in an agitated liquid–liquid dispersion," Chem. Eng. Sci., 30, No. 9, 1057-1064 (1975).
8. E. Barnea and J. Mizrahi, "On the 'effective' viscosity of liquid–liquid dispersions," Ind. Eng. Chem.: Fundam., 15, No. 2, 120-126 (1976).
9. F. Sherman (ed.), Emulsions [Russian translation], Khimiya, Leningrad (1972).
10. M. L. Mao and S. S. Marsden, "Stability of crude oil in water emulsions as a function of shear rate, temperature, and oil concentration," J. Can. Petroleum Technol., 16, No. 2, 54-59 (1977).
11. V. P. Tronov and A. K. Rozentsvaig, "Coalescence of the dispersed phase in fluid emulsions due to turbulent flow," Zh. Prikl. Khim., 49, No. 1, 231-232 (1976).